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Evaluation of Wellbore Performance using Productivity Index

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ABSTRACT: *The performance of petroleum wells, especially, the development wells is very relevant to the petroleum engineer in other to evaluate the target reservoir. To produce efficiently from production wells, it is important to evaluate the performance of the well both on future and present preferences. In this work a partial differential equation model was developed for slightly compressible fluids and was simulated using COMSOL Multiphysics. The analysis was performed using different reservoir configurations and the results showed that increasing the length of the horizontal drain-hole leads to increase in productivity index and well productivity, provided the flow rate is kept at an optimal value. Increasing the well production rate lowers the productivity index of a well; this is due to the increased pressure drop in the reservoir. From the analysis performed regarding the applicability of the model in production wells, the higher the beta-factor of the formation, the lower the productivity of the wells, also the results showed that non-Darcy flow regime could exist in any porous media as long as the pressure gradient was sufficient for high velocity flow. A comparative analysis of the calculated productivity index of all the geometries used in the numerical computation was done and the results showed that the highest productivity index for all the geometries was obtained when the β factor was assumed to be zero. Therefore, it was concluded that the horizontal drain hole has the highest calculated productivity index for all the cases due to the increased exposure of the horizontal drain-hole to the reservoir.*

KEY WORDS: *Well Performance; Mathematical Modelling; Pressure Gradient and Productivity Index*

I. INTRODUCTION

Despite the benefits of geophysical and geological methods during the exploration of crude oil and associated products, the only way to confirm the presence of petroleum fluids is to drill a well (Fidelis, 2014). In the development of petroleum fields, several wells are drilled for different purposes. The basic types of wells used include exploration wells, development wells and observation wells. Exploration wells are drilled purely for exploratory (information gathering) purpose in a new area, the site selection is usually based on seismic data, satellite surveys etc. A development well is a well drilled in a proven producing area for the production of oil or gas already proven by appraisal drilling to be suitable for exploration. An observation well is a well that is used to observe changes in groundwater levels over a period, or more specifically during a pumping test. The development wells which include production or injection are used for the transmission of fluids either from the reservoir to the surface (in the case of petroleum fluids or water production) or from the surface to the reservoir (in the case of injection of fluids for pressure maintenance). The efficiency of the transmission of these fluids is of great importance to the petroleum engineer in order to enable him know the behavior of the well. This behavior is what is general referred to as well performance and it is a phenomenon that depends on several factors and a proper understand of it will require adequate knowledge in some specific areas of petroleum engineering.

In as much as an apparent need has been felt for definitions and limitations concerning well-performance measurements, models, analysis and interpretations, are proposed to discuss in some detail those terms most commonly encountered in current production practice. Well performance basically means those sub-surface characteristics of the well which are not directly related to the method by which the fluid is brought to the surface (Kantzer & Trostel, 1997). Any production well is drilled and completed to move the oil and gas from its original location in the reservoir to the stock or sales line. The fluids must travel through the reservoir and the piping system and ultimately flow into a separator for gas-liquid separation. The production system or well can be simple or can include many components in which energy or pressure losses occur. The accurate design of oil and gas well tubing strings requires ability to understand the flow performance in the wells. Wells normally produce a mixture of oil and gas regardless of whether they are classified as oil wells or gas wells (James & Hemanta, 2003). The common occurrence in most wells shows that, the fluids entering the wellbore from the reservoir can range from an under saturated oil to a single-phase gas. Free water can accompany the fluids as a result of water coning, water flooding or production of interstitial water (Weller, 2001).

The reservoir remains the most important component in the total well system. If accurate and precise predictions and analysis are not made about the reservoir pertaining to what will flow into the borehole from the reservoir, it will be difficult or inappropriate to try to evaluate the performance of the well at any time. The relationship between the flow rate performance of a well and the pressure drawdown is a function of parameters such as rock properties, well components, fluid properties, flow regime, fluid saturations in the reservoir, compressibility of the fluids, possible formation damage or stimulation, drive mechanisms and turbulence in the region of flow. The flow from the reservoir into the well has been called inflow performance and a plot of producing rate versus buttomhole flowing pressure is called an inflow performance relationship (Dale, 2003)**.** The objective of this work is to develop mathematical models for well performance evaluation with a wide range of applicability in the petroleum industry.

II. MODEL DEVELOPMENT FOR WELL PERFORMANCE EVALUATION

The model developed in this work is a non-linear mathematical flow equation and it is derived using the combination of the three basic equations governing the principles of fluid flow in wells and other systems, which include:

- The Continuity Equation (law of conservation of mass)
- Equation of state (EOS)
- Law governing the dynamics of fluid flow (Newton's law)

The continuity equation, for the reservoir system where the well is drilled is given by:

$$\emptyset\frac{\partial\rho}{\partial t}+Div(\rho\vec{v})=0$$
(1)

Let us assume that the porosity of the reservoir system is constant, therefore after simplifying the continuity equation for the model becomes:

$$\emptyset\rho'\cdot\frac{\partial P}{\partial t}=-\rho Div(\vec{v})-\rho'\vec{v}\cdot\nabla P \qquad (2)$$

Applying Darcy-Forchheimer equation for non-laminar flow, in order to derive the equation of state used for the model, results

$$\frac{dp}{dx}=\frac{\mu}{k}v+\beta\rho v^2$$
(3)

Expressing equation (3) in its vector form and assuming that $\alpha=\frac{\mu}{k}$ we have:

$$\alpha\vec{v}+\beta\rho|\vec{v}|\vec{v}=-\nabla P$$
(4)

The equation of state for the reservoir system is given by the expression:

$$\rho'=\gamma^{-1}\rho \qquad (5)$$

Where $\rho=\rho_0 e^{\gamma^{-1}(P-P_0)}$

And γ^{-1} is the compressibility of the flowing fluid.

The vector velocity is a mono-directional term and is also a function of time. This vector velocity cannot be uniquely represented as a function of the pressure gradient, ∇P, we assume an approximation that is given by:

$$\vec{v}=\vec{v}_\beta=(v_1,v_2,v_3)=f_\beta(|\nabla P|)\nabla P$$
(6)

Substituting equation (6) into Darcy-Forchheimer equation (equation 4) gives:

$$\alpha\big(f_\beta(|\nabla P|)\nabla P\big)+\beta\rho\cdot f_\beta(|\nabla P|)\nabla P\cdot f_\beta(|\nabla P|)\nabla P\ =-\nabla P \qquad (7)$$

Re-arranging and factorizing equation (7) gives:

$$\nabla P[1+\alpha\left(f_\beta(|\nabla P|)\right)+\beta\rho\cdot\left(f_\beta(|\nabla P|)\right)^2=0 \qquad (8)$$

Solving equation (8) by the quadratic formula and taking on the positive root of the result gives:

$$f_\beta(|\nabla P|)=\frac{2}{\alpha+\sqrt{\alpha^2+4\beta\rho|\nabla P|})}$$
(9)

Equation (9) becomes solution of the velocity vector to the Darcy-Forchheimer equation of the model.
Substituting the Darcy-Forchheimer parameters (i.e. equation 6) into the continuity equation, since for slightly-compressible fluids, such as oil, the term $\rho'\vec{v}\cdot\nabla P$ is equal to zero, we have:

$$\frac{\partial P}{\partial t}=-\frac{\gamma}{\emptyset}\boldsymbol{Div}\left(f_\beta(|\nabla P|)\nabla P\right) \qquad (10)$$

Equation (10) is the form of the partial differential equation (PDE) that is used to model the non-linear Darcy-Forchheimer flow in any porous medium. The negative sign implies reduction in pressure gradient in the direction of flow. Further assuming a porosity value of 100% for fluid flow in pipes, since the bulk volume of the pipe is almost entirely open to flow, therefore the equation for flow through petroleum wells gives:

$$\frac{\partial P}{\partial t}=-\gamma Div\left(f_\beta(|\nabla P|)\nabla P\right) \qquad (11)$$

Therefore equation (11) is the partial differential equation (PDE) that was resolved in COSMOL to predict wellbore performance during production or other well operation activities.
The assumptions made in arriving at this equation are:

- Darcy's law and continuity equations are applied
- Non-Laminar flow
- Product of derivative terms that are not functions of time are negligible
- Compressibility is not equal to zero
- Pressure is independent of rock and fluid properties
- Homogeneous and isotropic porous medium with uniform thickness
- Constant well diameter
- Negligible gravitational forces
- Total porosity for flow through wells

III. RESULTS AND DISCUSSION

The results obtained from the simulation of the developed model using COSMOL Multiphysics, which is a software mainly for resolving partial differential equations are presented for three cases: Horizontal well in a rectangular reservoir, Circular well in a rectangular reservoir and Circular well in a circular reservoir. The discussions of these results follow immediately. The tabulated results and the reservoir parameters are presented in the appendices.

Horizontal Well in A Rectangular Reservoir : Geometry 1 used in the numerical computation of this model is a horizontal drain-hole in a rectangular reservoir with locations relative to the boundaries of the reservoir (generated using the simulator) as shown in figure 1. The dimensions used for the computation of the horizontal drain-hole are as shown in table 1 in appendix A1.

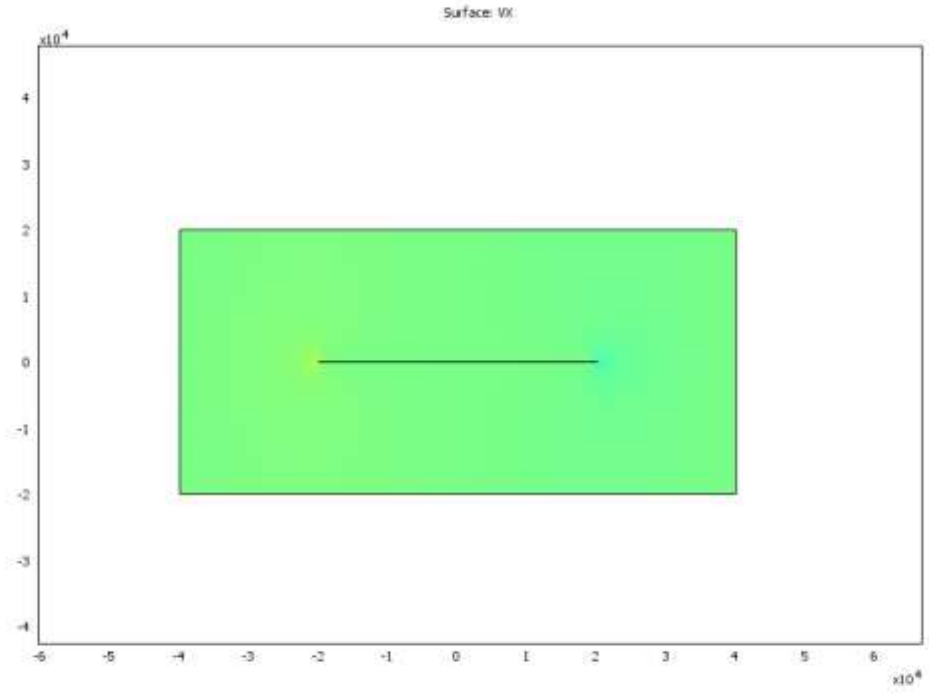

Figure 1: Geometry of the Horizontal Drain in a Rectangular reservoir

The results of the variation of the various calculated productivity index with length of the horizontal drain-hole at different β factor values in the reservoir are as shown graphically in figures; 2, 3 and 4 below and tables 2, 3 and 4 in Appendix A1.

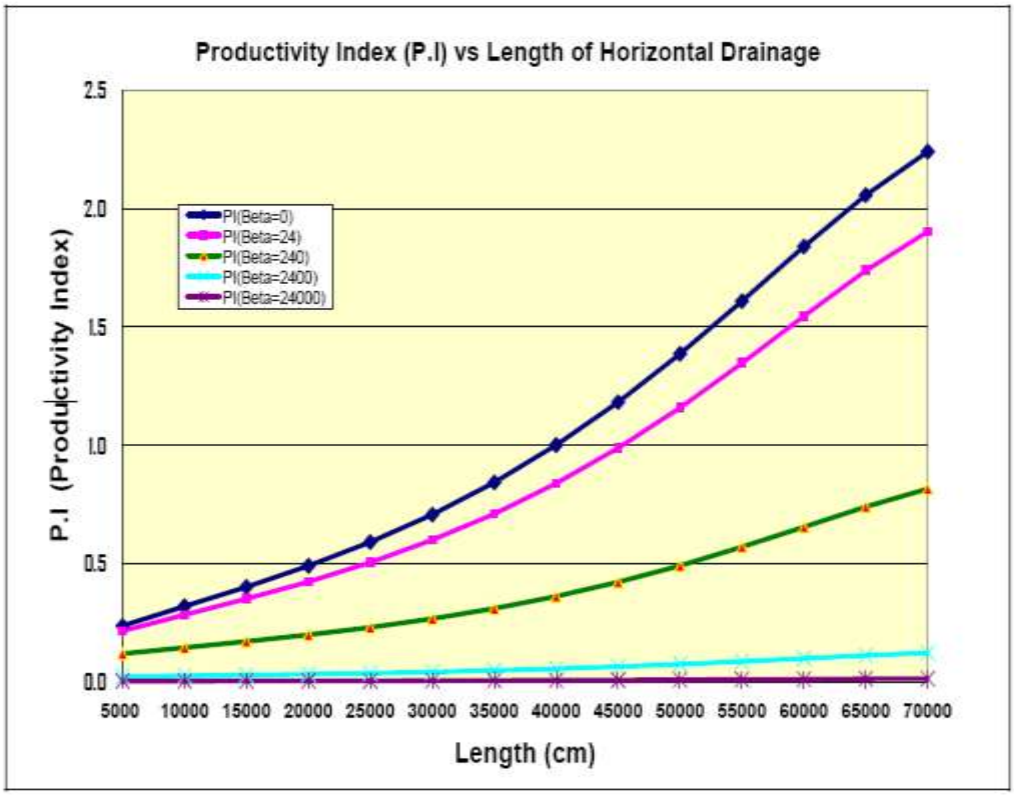

Figure 2: Plot of Productivity Index at different drain-hole lengths

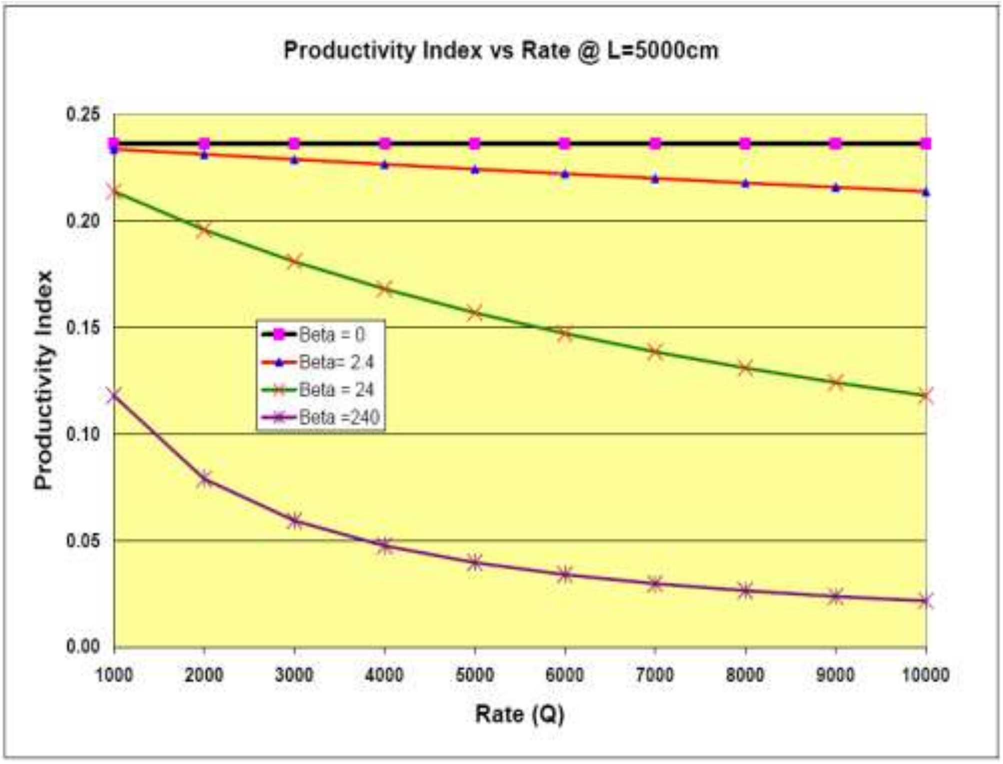

Figure 3: Productivity index versus rate @ L=5000 cm (164ft)

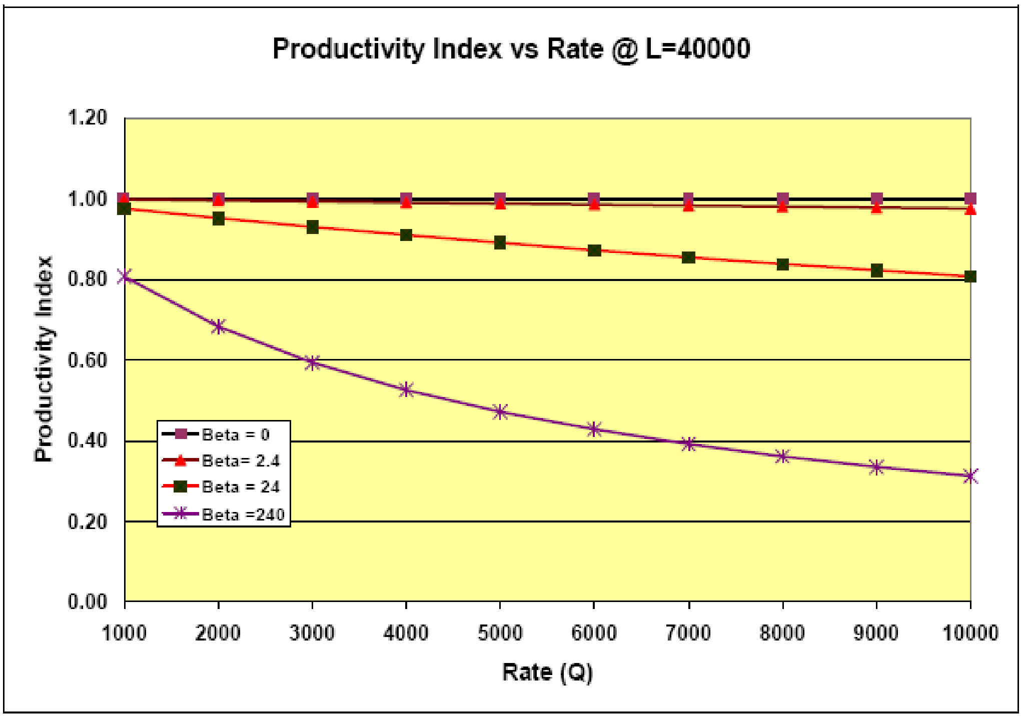

Figure 4: Productivity index versus rate @ L = 40,000 cm

Circular Well in A Rectangular Reservoir: Geometry 2 used in the numerical computation of this model is the circular drain-hole in a rectangular reservoir with locations relative to the boundaries of the reservoir (generated using the simulator) as shown in figure 5. The dimensions used for the computation of the vertical drain-hole are the same as the horizontal drain-hole shown in table 1 in Appendix A1.

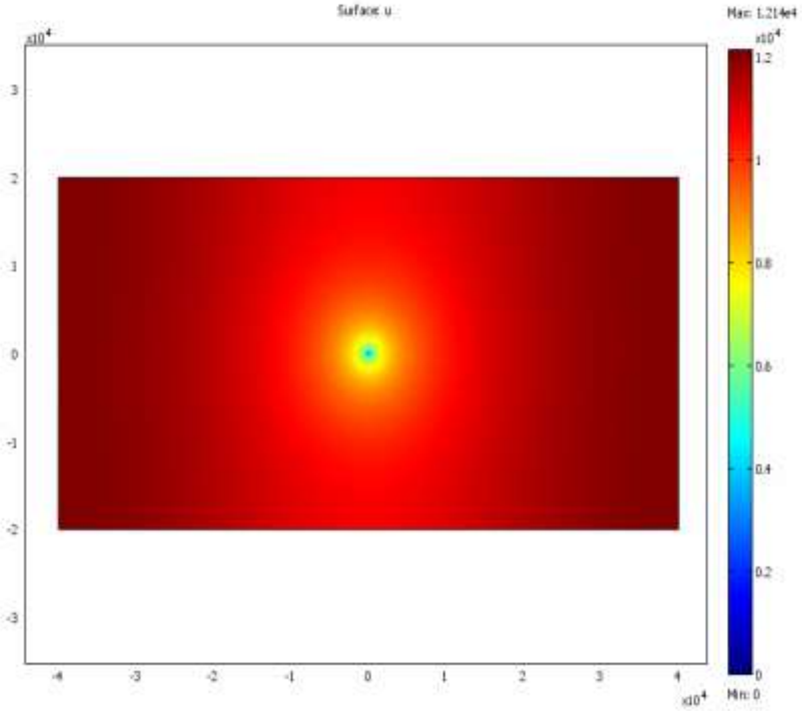

Figure 5: Geometry of the Circular well in a rectangular reservoir generated by the simulator

The results of the variation of the calculated productivity index with length of the circular drain-hole at different β factor values in the reservoir is as shown graphically in figure 6 below and table 5 in Appendix A1.

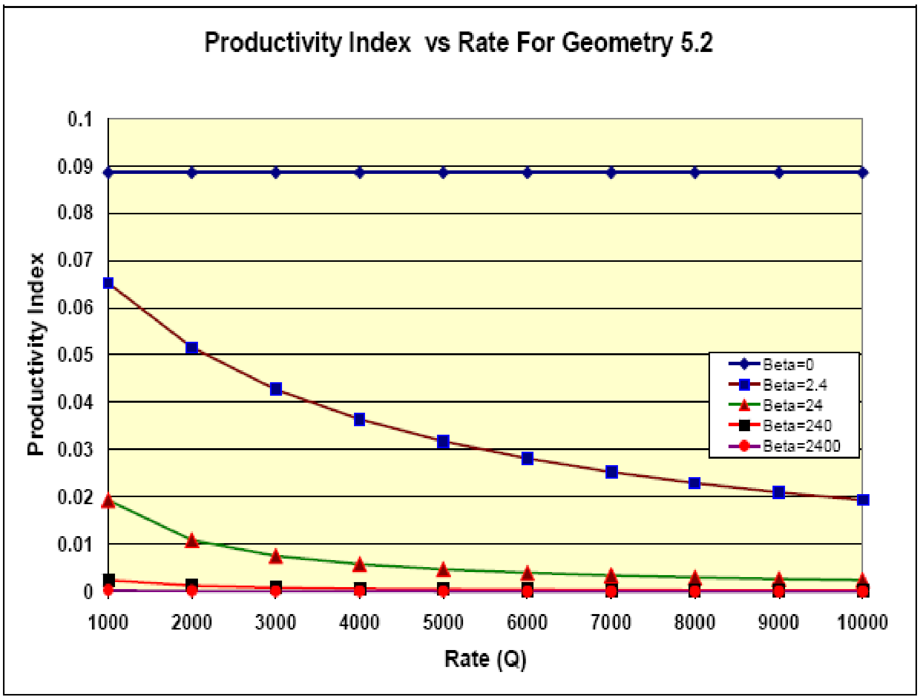

Figure 6: Productivity index plot for the circular well with a rectangular reservoir.

Circular Well Centered in A Circular Reservoir : Geometry 3 is a circular well in a circular shaped reservoir, the position of the well relative to the reservoir boundaries is as shown in figure 7. The dimension of the well and the reservoir is as shown in table 6 in Appendix A1.

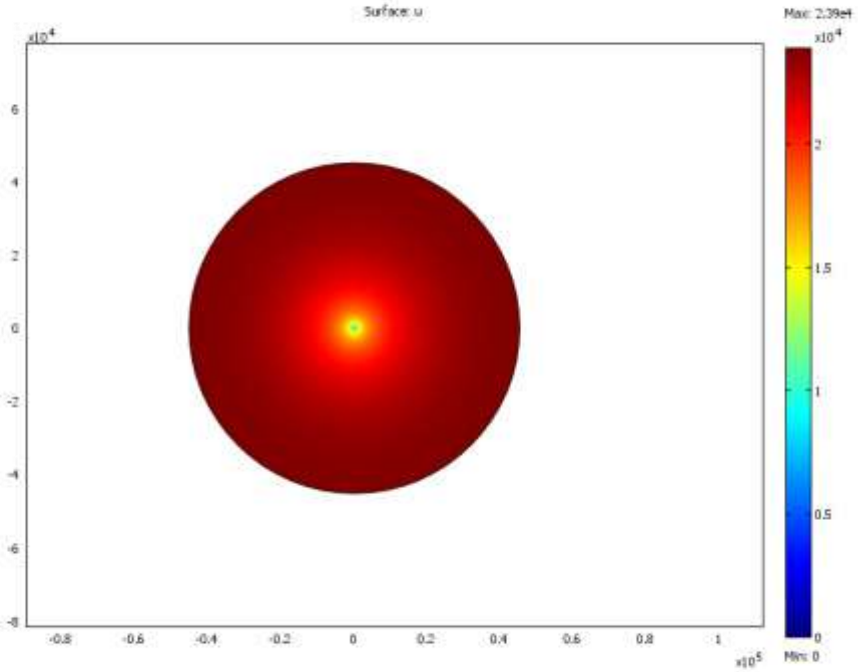

Figure 7: Circular well in a circular reservoir (Geometry 3)

The results of the variation of the numerical computations of productivity index with length of geometry 3 for the circular well centered in a circular reservoir at different β factor values as shown graphically in figure 8 below and table 6 in Appendix A1.

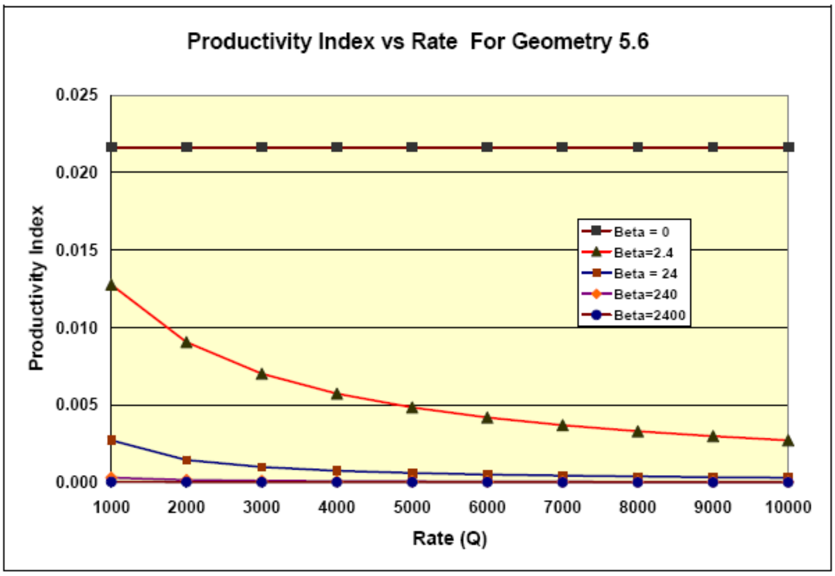

Figure 8: Productivity index plot for Geometry 3

IV. DISCUSSION OF RESULTS

A comparative analysis was done on the results of the numerical computation of the productivity index of the horizontal drain-hole, for different mix and combination of flow rate and β factor in the reservoir. Figures 2 and 3 show the variation of the productivity index of a horizontal drain-hole as β factor changes in the reservoir. A comparative analysis was done on the results of the numerical computation of the productivity indices of all the geometries used in evaluating the wellbore performance for different types of wells with different locations in the reservoir. For the case of Darcy flow in the reservoir, the results showed that the highest productivity index for all the geometries were obtained when the β factor was assumed to be zero, as shown in the tables in

Appendix A1.However, when non-Darcy (non-linear) flow is assumed in the reservoir, there is a substantial drop in the productivity index for all the geometries, which increase as the β factor increases for all the cases. This productivity drop is due to the increased pressure drop in the reservoir, due to the increased dissipation of energy in the porous media as the β factor increases. As expected, the horizontal drain hole has the highest calculated productivity index for all the cases due to the increased exposure of the horizontal drain-hole to the reservoir. The location of a well in the reservoir is important to the productivity of the reservoir, this is why adequate geological and reservoir evaluation are done before determining the location and placement of a well in the reservoir.

V. CONCLUSION

Results obtained from this work showed that;

- The beta factor is a flow rate phenomenon, directly influenced by the magnitude of the flow rate in the medium.
- Increasing the length of the horizontal drain-hole leads to increase in productivity index and well productivity, provided the flow rate is kept at an optimal value.
- Increasing the well production rate lowers the productivity index of a well; this is due to the increased pressure drop in the reservoir.
- There is an optimum rate at which wells should be produced to obtain maximum production from the wells and efficiently utilize the natural reservoir energy.
- Selection and usage of beta factor correlations may be misleading; experimental determination of beta factor based on core analysis will be more accurate.
- The higher the beta-factor of the formation, the lower the productivity of the well.
- Results show that non-Darcy flow regime can exist in any porous media as long as the pressure gradient is sufficient for high velocity flow.
- The main determining property for non-linearity in flow is the permeability of the porous media.

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APPENDIX A1

Table 1 Dimension of the horizontal well and horizontal reservoir ((ABIODUN, 2007)

Parameter	Unit/Symbol	Value
Length	L, ft (m)	244 (800)
Well radius	r_w, ft (cm)	0.5 (15)
Width	w, ft (m)	122 (400)

Table 2: Productivity Index at different drain-hole lengths

			Productivity Index at Different drain-hole lengths				
Length (cm)	Q(cm^3/s)	Q (ft^3/day)**	β=0	β=24	β=240	β=2400	β=24000
5000	1000	100	0.23639	0.21403	0.11814	0.02169	0.00237
10000	2000	200	0.31863	0.28256	0.14406	0.02460	0.00265
15000	3000	300	0.40047	0.34989	0.16974	0.02786	0.00298
20000	4000	400	0.48950	0.42256	0.19768	0.03161	0.00336
25000	5000	500	0.59029	0.50444	0.22939	0.03599	0.00382

30000	6000	600	0.70670	0.59888	0.26623	0.04117	0.00436
35000	7000	700	0.84233	0.70906	0.30960	0.04734	0.00500
40000	8000	800	1.00012	0.83789	0.36082	0.05470	0.00577
45000	9000	900	1.18190	0.98748	0.42102	0.06341	0.00668
50000	10000	1000	1.38673	1.15807	0.49071	0.07354	0.00774
55000	11000	1100	1.60952	1.34669	0.56924	0.08502	0.00895
60000	12000	1200	1.83912	1.54509	0.65409	0.09750	0.01025
65000	13000	1300	2.05735	1.73806	0.73962	0.11023	0.01159
70000	14000	1400	2.24032	1.90292	0.81515	0.12164	0.01279

Table 3: Productivity index @ L = 5,000cm at different rates and β values

L=5000cm(164ft)	**Productivity Index at Different Beta Values**				
Q (cm3/s)	**Q (ft3/day)**	**β=0**	**β=2.4**	**β=24**	**β=240**
1000	100	0.2364	0.2338	0.2140	0.1181
2000	200	0.2364	0.2314	0.1960	0.0790
3000	300	0.2364	0.2290	0.1810	0.0594
4000	400	0.2364	0.2267	0.1682	0.0476
5000	500	0.2364	0.2245	0.1570	0.0397
6000	600	0.2364	0.2223	0.1473	0.0340
7000	700	0.2364	0.2201	0.1387	0.0298
8000	800	0.2364	0.2180	0.1311	0.0265
9000	900	0.2364	0.2160	0.1243	0.0238
10000	1000	0.2364	0.2140	0.1181	0.0217

Table 4 Productivity index @ L = 40, 000cm at different rates and β values

L=40,000cm(164ft)	**Productivity Index at Different Beta Values**				
Q (cm3/s)	**Q (ft3/day)**	**β=0**	**β=2.4**	**β=24**	**β=240**
1000	100	1.0001	0.9975	0.9748	0.8067
2000	200	1.0001	0.9948	0.9515	0.6829
3000	300	1.0001	0.9922	0.9298	0.5934
4000	400	1.0001	0.9897	0.9094	0.5253
5000	500	1.0001	0.9871	0.8901	0.4714
6000	600	1.0001	0.9846	0.8718	0.4276
7000	700	1.0001	0.9821	0.8544	0.3913
8000	800	1.0001	0.9796	0.8378	0.3608
9000	900	1.0001	0.9772	0.8219	0.3347
10000	1000	1.0001	0.9746	0.8067	0.3121

Table 5: Productivity Index at various rate and β values

	Productivity Index at Different Beta Values					
Q (cm3/s)	**Q (ft3/day)**	**β=0**	**β=2.4**	**β=24**	**β=240**	**β=2400**
1000	100	0.08853	0.06520	0.01934	0.002407	2.47 x 10^{-4}
2000	200	0.08853	0.05160	0.01085	0.00122	1.24 x 10^{-4}
3000	300	0.08853	0.04269	0.007544	8.17 x 10^{-4}	8.24 x 10^{-5}
4000	400	0.08853	0.03641	0.005781	6.14 x 10^{-4}	6.18 x 10^{-5}
5000	500	0.08853	0.03174	0.004686	4.92 x 10^{-4}	4.95 x 10^{-5}
6000	600	0.08853	0.02813	0.00394	4.10 x10^{-4}	4.12 x 10^{-5}
7000	700	0.08853	0.02526	0.003399	3.52 x 10^{-4}	3.53 x 10^{-5}
8000	800	0.08853	0.02292	0.002988	3.08 x10^{-4}	3.09 x 10^{-5}
9000	900	0.08853	0.02098	0.002666	2.74 x 10^{-4}	2.75 x 10^{-5}
10000	1000	0.08853	0.01934	0.002407	2.47 x 10^{-4}	2.47 x 10^{-5}

Table 6 Dimension of the horizontal well and reservoir ((ABIODUN, 2007)

Parameter	Unit/Symbol	Value
Radius of Reservoir	r_e, ft (m)	1480 (451.35)
Well radius	r_w, ft (cm)	0.5 (15)

Table 7: Productivity Index at various rate and beta values

	Productivity Index at Different Beta Values					
Q (cm3/s)	**Q (ft3/day)**	**β=0**	**β=2.4**	**β=24**	**β=240**	**β=2400**
1000	100	0.021646	0.012771	0.002724	3.07×10^{-4}	3.11×10^{-5}
2000	200	0.021646	0.009058	0.001453	1.55×10^{-4}	1.56×10^{-5}
3000	300	0.021646	0.007018	9.91×10^{-4}	1.03×10^{-4}	1.04×10^{-5}
4000	400	0.021646	0.005728	7.52×10^{-4}	7.76×10^{-5}	7.79×10^{-6}
5000	500	0.021646	0.004839	6.06×10^{-4}	6.21×10^{-5}	6.23×10^{-6}
6000	600	0.021646	0.004188	5.07×10^{-4}	5.18×10^{-5}	5.19×10^{-6}
7000	700	0.021646	0.003692	4.36×10^{-4}	4.44×10^{-4}	4.45×10^{-6}
8000	800	0.021646	0.003301	3.83×10^{-4}	3.89×10^{-4}	3.89×10^{-6}
9000	900	0.021646	0.002985	3.46×10^{-4}	3.46×10^{-4}	3.46×10^{-6}
10000	1000	0.021646	0.002724	3.07×10^{-4}	3.11×10^{-5}	3.12×10^{-6}