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Modeling Heat Transfer Coeficient Of Air Using Buckingham Pi-Theory

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***ABSTRACT**: Heat transfer coefficients of dryers are useful tools for correlation formulation and performance evaluation of process design of dryers as well as derivation of analytical model for predicting drying rates. A model equation for predicting heat transfer coefficient of air in a batch dryer using Buckingham Pi-theorem and dimensional analysis at various velocities has been formulated. The model was validated by drying unripe plantain chips in a batch dryer at air velocities of 0.66, 0.92 and 1.20m/s at temperatures of 42, 54.45 and 66°C, respectively. Based on the analogy of heat and mass transfer rate equations for constant drying period, the prediction from the developed model reasonably agreed with the experimental data.*

KEYWORDS : *Batch Dryer, Heat Transfer Coefficient and Model Equation*

I. INTRODUCTION

Drying is a kinetic process that involves the removal of liquid, usually water from a moist material: solid, liquid or gas. The use of heat to remove liquid distinguishes drying from mechanical methods of removing water, such as: centrifugation, decantation, sedimentation and filtration in which no change in phase from liquid to vapour is experienced. The application of heat to remove moisture can also be found in processes like distillation and absorption. In the drying of solids to remove water a specialized device called dryer is used, and the desirable end products are in solid form. The final moisture contents of the dried solids are usually less than 1%. The chemistry of drying a moist material can be represented as:

$$\text{Moist material} \longrightarrow \text{solid} + \text{vapor} \qquad (1.1)$$

When the heat transfer by pure convection is used to dry a wet solid, the heat supplied is solely by sensible heat in the drying gas stream. A dynamic equilibrium exist between the rate of heat transfer to the material and the rate of vapor (mass) removal from the surface at instance, (that is, drying rate) and may be represented as follows.

$$\frac{dx}{dt} = hA^1 \frac{\Delta T}{\lambda} \qquad (1.2)$$

The area of the heat and mass transfer may be assumed to be approximately equal [1]. The study of convective heat transfer is centered on ways and means of determining the heat transfer coefficient, h for various flow regions (laminar, transition or turbulent flow) and over various geometries and configurations. The local and average heat transfer coefficient may be correlated by equations (1.3) and (1.4) respectively

$$Nu_x = f\ (x^*,\ \mathrm{Re}_x,\ P_r) \qquad (1.3)$$

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where the subscript x emphasize the condition at a particular location on the surface. The problem of convection involves how these functions are obtained, there are two approaches: theoretical and xperimental. Theoretical approach involves solving the boundary layer equation for a particular geometry and equation such as equation (1.5)

$$Nu_x = \frac{hL}{k} = \pm \frac{\partial T^*}{\partial y^*} 1_{y^*} = 0 \qquad (1.5)$$

which is a dimensionless temperature gradient at the surface. In the experimental approach, for a prescribed geometry in a parallel flow, if heated, convection heat transfer coefficient which is an average associated with the entire system could then be computed from Newton's law of cooling. And from the knowledge of the characteristic length and the fluid properties, the Nusselt, Reynolds and Prandtl numbers could be computed from their definitions. Meanwhile, the relevant dimensionless parameters for low-speed, forced convection boundary layer have been obtained by non-dimensionalizing the differential equation that describes the physical process occurring within the boundary layer. An alternative approach is the use of dimensional analysis in the form of BuckingHam Pi theorem. The success of the theorem depends on the ability to select from intuition the various parameters that influence the problem. Therefore, knowing that

$$h \;=\; f \;\; (K, C_p \;\; \rho, \, \mu, V, \, L) \tag{1.6}$$

Before hand one could use the BuckingHam Pi theorem to obtain h, in equation (1.6), [2]

II. MODELLING AND EXPERIMENTAL VALIDATION

MODEL FORMATION: To successfully create non-dimensional groups, each time a need arises, a set of rules must be followed [3]; the Raleigh method and Buckingham's Pi-Theorems are reliable. The Raleigh method is an elementary technique for finding a functional relationship between variables. Although very simple, the method does not provide any information concerning the number of dimensionless group that can be obtained. Another drawback of the method is that it can only be used for the determination of the expression for variables that depend on a maximum of three or four independent variables only.The BuckingHam Pi theorem is basically an improvement over the Raleigh's method. Apart from its advantage of being able to handle large sets of variables, it gives a ready clue on how many dimensionless groups are designated by Pi.

$$f\,(a, \, h, K, D, \, \mathrm{Re}, \;\, P_r) \;\; = 0 \tag{2.1}$$

$$\mathrm{Re} \;\; = \;\; \frac{\upsilon VD}{\mu} \tag{2.2}$$

$$\mathrm{Pr} \;\; = \;\; \frac{\mu Cp}{k} \tag{2.3}$$

Therefore

$$f\,\{a \;\; (h, Kv, D, \; \rho, V, \mu, \; Cp)\} \;\; = 0 \tag{2.4}$$

where a. is a constant

Choosing M, L, T, and K as fundamental dimension for mass, length, time and temperature, implies that:

Number of Fundamental Dimension, m = 4

Number of Quantities, n = 7

Therefore

$$N\eta \;\; = \; 7 - \; 4 \; = \; 3$$

Since m is 4, there will be four repeating quantities: Geometric (D). Flow (v), fluid density (ρ) and specific heat capacity (Cp)

Table 2.1: Dimension of Quantities

S/No	Quantity	Symbol	Unit	Dimension
1	Diameter	D	M	L
2	Density	p	Kgm^{-3}	ML^{-3}
3	Velocity	V	m/s	LT^{-1}
4	Viscosity (absolute)	i	$kgm^{-1}s^{-1}$ kgms^{-}	$ML^{-1}T^{-1}$
5	Thermal Conductivity	K	$^{3}K^{-1}$ $m^2s^2K^{-1}$	$MLT^{-3}K^{-1}$
6	Heat Capacity	C	$kgs^{-3}K^{-1}$	$L^2T^{-2}K$
7	Heat Transfer Coefficient	H		$MT^{-3}K^{-1}$

π – terms

$$\pi_1 \;\; = \;\;\;\; D^{w1} V^{x1} \; \rho^{y1} \; Cp \; z^1 K \tag{2.5}$$

$$\pi_2 \;\; = \;\;\;\; D^{w2} V^{x2} \; \rho^{y2} \; Cp \; z^2 \; \mu \tag{2.6}$$

$$\pi_3 = D^{w3} V^{x3} \rho^{y3} Cp\ z^3 h \qquad (2.7)$$

Setting up and solving equations for π groups, using the dimensions in table 2.1 gives

For $\pi_1 = D^{w1} V^{x1} \rho^{y1} Cp\ z^1 K_v$

Putting this dimension terms, gives

π_1: $(LW^1)(LT^{-1})^{X1}(ML^{-3})^{Y1}(L^2T^{-2}K^{-1}) = M^0L^0T^0K^0$ (2.8)

By equating the powers of the dimensions on both sides of equation (2.8), we have

M: $Y_1 + 1 = O$ $Y_1 = -1$ (2.9)
L: $W_1 + X_1 - 3Y_1 + 2Z_1 + 1 = 0$ (2.10)
T: $-X_1 - 2Z_1 - 3 = O$ (2.11)
K: $-Z_1 - 1 = 0$ (2.12)

Substituting (2.12) into (2.11)

$-X_1 - 2(-1) - 3 = O$ (2.13)
$-X_1 + 2 - 3 = O$ $X_1 = -1$ (2.14)

Substituting (2.9), (2.12) and (2.14) into (2.10)

$W_1 - 1 - 3(-1) + 2(-1) + 1 = 0$ (2.15)
$W_1 + 1 - 3 - 2 + 1 = 0$ $W_1 = -1$ (2.16)

Hence,

$$\pi_1 = D^{w1} V^{x1} \rho^{y1} Cp\ z^1 K \qquad (2.17)$$

Or $\pi_1 = \dfrac{K}{\rho VDCp}$ (2.18)

Using similar procedure for equations (2.6) and (2.7), we obtain values for π_2, π_3, as in (2.19) and (2.20) respectively.

Hence $\pi_2 = \dfrac{\mu}{\rho VD}$ (2.19)

and

$$\pi_3 = \frac{h}{\rho V C_P} \qquad (2.20)$$

Combining equation (2.18), (2.30) and (2.39), we have

$$f\left\{a\left(\frac{kv}{\rho VDCp}, \frac{\mu}{\upsilon VD}, \frac{h}{\rho VCp}\right)\right\} = 0 \qquad (2.21)$$

Hence $\dfrac{h}{\rho VCp} = a\left(\dfrac{\mu}{\rho VCp}, \dfrac{kv}{\rho VDCp}\right)$ (2.22)

Using h as the dependent variable and V the independent variable and applying equation 2.18,2.19 and 2.20 to obtain the values of π_1, π_2 and π_3.

Therefore, from equation 2.22, $\pi_3 = a(\pi_{2,,} \pi_1)$ (2.23)

Plotting the independent function (π_3) against the dependent function (π_1), (π_2), we obtain graphs which when regressed yields the theoretical heat transfer coefficient (h_B).

MODEL VALIDATION

EXPERIMENTAL MATERIALS AND METHOD: Unripe plantains purchased from a local market, Port Harcourt, were used in the experiment. The moisture content of the plantain before drying was 26.76g dry base. The moisture content was determined manually by periodically weighing of the sample at 3. minutes interval, for three hours (61 data points). According to [2], for evaporative heating based on heat and mass transfer analogy, as the gas flow over the moist material evaporation occur from the surface, and the energy associated with the phase change is the latent heat of vaporization of the liquid.

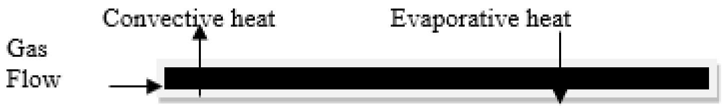

Fig. Heat and Mass Transfer Analogy

Applying conservation of energy to a control surface about the material

$$q''_{convective} + q''_{added} = q''_{evaporation} \quad (2.24)$$

Where $q''_{evaporation}$ may be approximated as the product of the mass (moisture loss) and the latent heat of vaporization.

$$q''_{evaporation} = nA\,h_{fg} = \frac{dx}{dt}\lambda \quad (2.25)$$

$$q''_{convective} = hA\,(T_{hf} - T_s) \quad (2.26)$$

Since no heat is added, and for constant drying

$$q''_{evaporation} = q''_{evaporation} \quad (2.27)$$

Therefore

$$hA\,(T_{hf} - T_s) = hA(\Delta A) = \frac{dx}{dt}\lambda \quad (2.28)$$

Where $\frac{dx}{dt}\lambda$ represents the constant drying rate

For surface temperature, $T_s = 28^0C$

and the heating fluid is at T_5, $T_{hf} = T_5$

$$T_5 - T_S = \Delta T_5 \quad (2.29)$$

Therefore $(h_E A\Delta T)_5 = \lambda \frac{dx^1}{dt^{(5)}}$ (2.30)

$$h_{E(3)} = \left(\frac{\lambda\,dx}{A(\Delta T)dt}\right)3 \quad (2.31)$$

When heating fluid is at T_1, $T_{hf} = T_1$ (2.32)

$$h_{E(1)} = \left(\frac{\lambda\,dx}{A(\Delta T)dt}\right)1 \quad (2.33)$$

Total Area of Plantain Chips A = $\left(\frac{\pi D^2}{4}\right)1$ (2.34)

D = diameter of Plantain Chips = 0.033m

For the 6 pieces of plantain = 0.198m

Hence A = $\left(\frac{\pi 0.198^2}{4}\right) = 0.030795m^2$

From equation 3.79, 3.80 and 3.81, the mean experimental heat transfer coefficient

$$h_E = \frac{\left\{\frac{\lambda}{A}\left(\frac{dx}{(\Delta T)dt}\right)_1 + \left(\frac{dx}{(\Delta T)dt}\right)_3 + \left(\frac{dx}{(\Delta T)dt}\right)_5\right\}}{3} \qquad (2.35)$$

III. RESULTS AND DISCUSSION

The results obtained from this work are presented in Table 3.1-3.5 and Figure 3.1-3.5

Results

Table 3.1: Calibration of the Batch Dryer

Number of turns of Knob (Knob rotation) Temperature(^{0}C)	Velocity(m/s)						
	V_1	V_2	V_A	V_B	T1	T_2	T_{av}
0(0^0)	3.60	3.40	3.50	0.66	66.00	66.00	66,00
3 (1080^0)	4.20	4.20	4.20	0.79	60.15	60.05	60.10
6(2160^0)	5.00	4.80	4.90	0.92	54.50	54.40	54.45
9 (3240^0)	5.65	5.75	5.70	1.07	48.30	48.20	48.25
12 (4320^0)	6.40	6.40	6.40	1.20	42.00	42.00	42.00

Where V and T are the average values

Substituting the thermo-physical properties and average velocities based on the calibration, into the Dittus-Boelter equation

$$hD = \frac{kv}{D}\, 0023\mathrm{Re}_D^{\;0.8}\ \mathrm{Pr}^{0.4}, \qquad (3.1)$$

we obtain values of heat transfer coefficient shown in Table 3.1.

Table 3.2: Thermo-physical Properties and Calculated Values of Heat Transfer Coefficient using Dittus-Boelter

Temperature (°C)	66.00	60.10	54.45	48.25	42.00
Velocity (m/s)	0.66	0.79	0.92	1.07	1.20
Density (kg/m^3)	1.0383	1.0516	1.0646	1.0749	1.0982
Specific Heat Capacity (m^{2}s-^{2}K^{-1})	1008.3	1008.2	1008.0	1007.8	1008.5
Dynamic Viscosity (kgm^{-1}s^{-1})	202.IE	200.18E-7	198.4E-7	196.9E-7	193.6E-7
Reynolds Number (-)	10070.64	12325.75	14661.86	17348.53	20216.86
Prandtl Number (-)	0.702	0.7022	0.7028	0.703	0.705
Heat Transfer Coefficient (Wm^{-2}K^{-1})	25.5297	30.0116	34.4787	39.4635	44.6411

Modelled Result: The values of the variables of table 3.2 are used to obtain the values of π_1, π_2, π_3, as shown in Table 3.3.

Table 3.3: $\pi -$ Groups

S/No	1	2	3	4	5
π_1	0.001121	0.0009245	0.0007842	0.0006681	0.0005831
π_2	0.0000993	0.0000811	0.0000696	0.0000572	0.0000495
π_3	0.03696	0.03583	0.03492	0.03402	0.03361

Experimental Result: Using the various air flow velocities experiments were performed in order to obtain the gradient for drying the unripe plantain. The experimental results have been shown in figures 3.3 to 3.5 where the graphs of moisture content versus time at v_1, v_3, and v_5 are shown below.

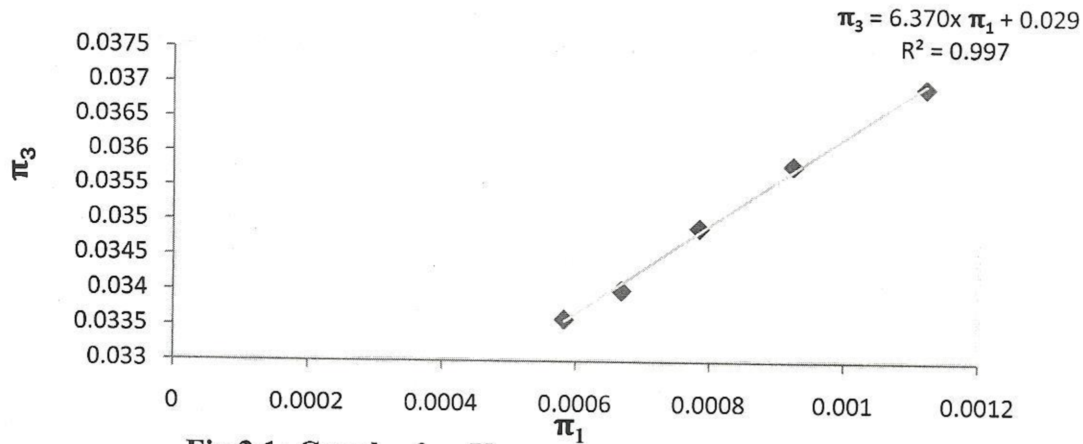

Fig 3.1: Graph of π_3 Versus π_1

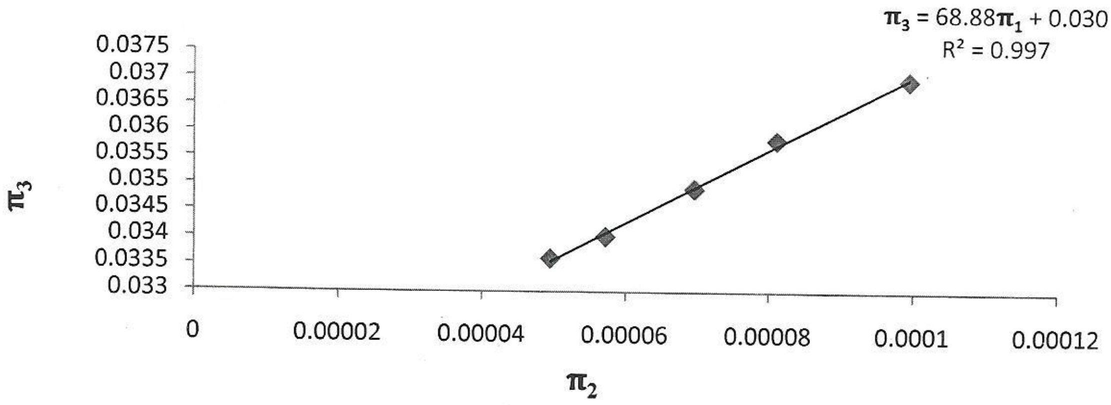

Fig. 3.2: Graph of π_3 versus π_2

The drying rate (evaporative heat) which is a product of evaporative flux and latent heat of vaporization obtained from Figure 3.3, 3.4 and 3.5, and the change in temperature based on v_1, v_2 and v_3 were used to obtain the experimental heat transfer coefficient as illustrated below:

$T_{hf(1)}$ = 42.00^{0}C, hence ΔT = 14.00^{0}C
$T_{hf(3)}$ = 54.45^{0}C, hence ΔT = 26.45°C
$T_{hf(5)}$ = 66.00^{0}C, hence ΔT = 38.00^{0}C
λ = Latent Heat of Vaporization of water = 250 1KJ/Kg [4]

$$\frac{dx}{dt}(1) = -0.176 g/\min = 0.00000285 Kg/s$$

$$\frac{dx}{dt}(3) = -0.173 g/\min = 0.00000288 Kg/s$$

$$\frac{dx}{dt}(5) = -0.171 g/\min = 0.00000293 Kg/s$$

$$h_E = \frac{2501000}{3x0.030795}\left(\frac{0.00000285}{14.00} + \frac{0.00000288}{26.45} + \frac{0.00000293}{38.00}\right)$$

$$h_E = 31.6379\,Wm^{-2}K^{-1}$$

Model Heat Transfer Coefficient (h_B) = 34.8524Wm^{-2}K^{-1}
Experimental Heat Transfer Coefficient (h_E) = 31.679Wm^{-1}K^{-1}

$$\%\ \text{error} = \frac{34.8524 - 31.6379}{34.8524}\ x\,100\% = 9.22\%$$

Therefore, a good agreement of about 91% was achieved.

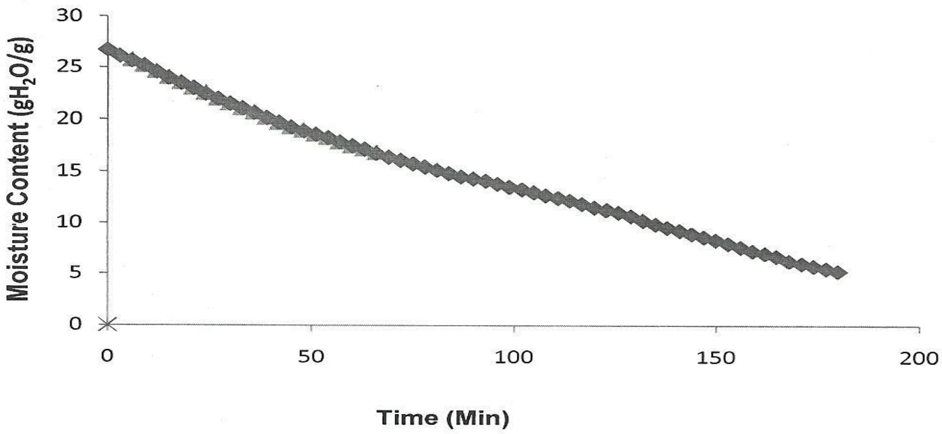

Fig. 3.3: Moisture Content versus Time (at V_1)

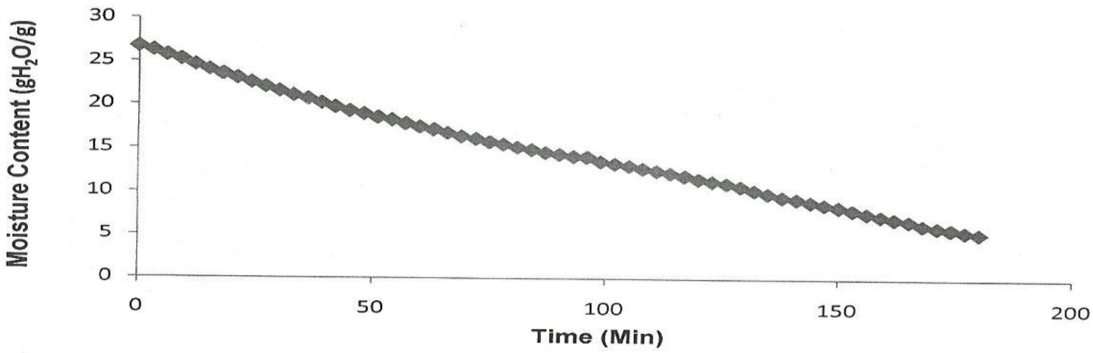

Figure 3.4: Moisture Content Versus Time (at V_3)

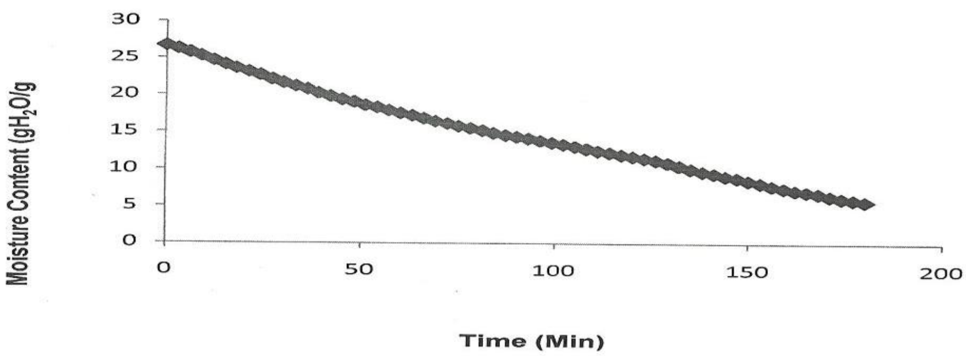

Figure 3.5: Moisture Content Versus Time (at V_5)

IV. DISCUSSION OF RESULTS

The values of the velocities and temperatures from the calibration and values from the thermo-physical table obtained from literature were substituted into the Dittus-Boelter equation to obtain the relationship between heat transfer coefficients (dependent variables) and velocities (independent variables), which form the basis for the Bucking Ham's Pi-Theorem.

The BuckingHam's Pi-Theorem, which uses dimensional analysis, was then used to obtain the Pi groups (π_1, π_2, π_3). Based on the Pi groups obtained, plots of π_3 versus π_1 and π_3 versus π_2 were obtained and regressed, using Microsoft Excel to obtain the modeled equations. The experiment when carried out at velocities of 0.66, 0.92 and 1.20 ms^{-1}, it was observed that the moisture in unripe plantain evaporated faster to approach dryness in the order; 0.66ms', 0.92 ms^{1} and l.2ms^{-1}. This is because higher velocities pickup more moisture from the system. From the plots of Figures 3.3, 3.4 and 3.5, the equations at the constant drying periods at various velocities were also obtained. Finally, a comparison of heat transfer coefficients obtained from the theoretical Buckingham Pi Theorem (modeled) and that obtained from the experimental result in the range of velocities investigated showed minimal variation of less than 10%

V. CONCLUSION

The heat and mass transfer analogy from Newton law of cooling has been shown to be a reliable correlation for obtaining heat transfer coefficient experimentally; also proven is that the Buckingham Pi-Theorem is a good and simplified method of obtaining correlation from experimental results. The comparison of both the experimental and model results shows a percentage error of 9.22%. The variation of the experimental and model heat coefficients could be as a result of experimental errors, although it is within acceptable level.

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